IOSR Journal Of Humanities And Social Science (IOSR-JHSS)
Volume 21, Issue12, Ver. 6 (December. 2016) PP 13-19
e-ISSN: 2279-0837, p-ISSN: 2279-0845.
www.iosrjournals.org

Investigating 11th Grade Students' Van-Hiele Level 2Geometrical Thinking

Ural, Alattin[1]
Mehmet AkifErsoy University,

ABSTRACT:- This study aimed to investigate 11th grade students' Van-Hiele level 2 of geometrical thinking. A total of 50 11th grade students participated in the study. The data collection tool included four open-ended and eight multiple-choice questions compatible with Van-Hiele level 2 of geometrical thinking. The study was a qualitative research. The success rates are those: defining geometric figures (62%), ordering the geometric figures (66%), having the knowledge of which geometrical property determines which geometric property (56%) or figure (61%). The errors sources basically are those: inadequacy in defining the geometric figures, deciding by looking at the figure instead of geometrical thinking, and not knowing the logic of ordering the figures.
***Keywords**: Geometry, geometrical thinking, Van Hiele.*

I. INTRODUCTION

The most significant one of the studies carried out on geometric thinking and how geometric thinking has evolved is Van Hiele model. This theory has been developed by Van Hiele since the1950's and has attracted interest in various studies worldwide. Most of the research studies (Burger & Shaughnessy, 1986; De Villiers &Njisane, 1987; Senk, 1983; Usiskin, 1982) show that Van Hiele theory can explain students' geometric thinking. The most important feature of Van Hiele model is that it defines the development of geometric thinking at five interrelated levels. Each of these five levels addresses to the thinking processes used in geometric contexts. These levels describe the way of thinking and the types of geometric ideas dealt with rather than focusing on how much information is known. The main difference between any two levels is the thinking objects; that is the concepts that can be understood geometrically. Van Hielemodel, is one which guides a teacher's practices in class and explains the difficulties encountered by students in geometry. According to this model, the factor which affects the development of geometric thinking is not age but it is experience in geometry. The geometric thinking levels of the model have the following properties (Duatepe, 2000; Hiele, 1986; Hoffer, 1983; Kılıç, 2003; OlkunveToluk, 2003; Van de Walle, 2004):

The focus on the visualization level (Level 0) is on recognizing and naming geometric objects. Children at this level perceive geometric shapes and objects holistically. The student identifies, names and compares shapes based on their appearance. Since appearance is dominant at this level, appearances might suppress the properties of a shape. A child at this stage cannot explain the properties of a shape and order geometric properties. The level 0 includes activities suitable for the 1st, 2nd and 3rd grader at primary school.

Students at Level 1 (analysis stage, analytic era) start analyzing the properties of shapes and can explain them completely; however, they cannot recognize the relations among shape classifications (order of properties). Murray (1996, 1997) states that students at this stage can explain the meaning of the terminology and symbols and produce their own definitions. A rectangle is known as a shape with four right angles, equal vertices and equal opposite sides. Generally, the primary level students are at stage 0 but transition into Level 1 activities can be initiated as of the 4th and 5th grades. Properties pertaining to geometric shapes are proven via experimental ways. A student at this stage compares shapes in terms of their parts and properties. Students at this level define a shape by saying many properties of a shape they know. They can make experimentally and intuitively some deductions such as which geometric property determines which geometric property.

Level 2 (ordering, informal deduction) is the stage at which the ability to establish interrelations among figure classes. Thinking productions at this level are properties of figures and the relationships among these properties. Students at this stage can understand the role of the definitions and relationships within figures and among properties of figures. They can order and group figures by their properties. The properties that describe a shape are known. Geometric shapes can be classified with sentences "If …then …" which require reasoning. Students can discuss the sufficient and necessary conditions for the definition of a geometric figure at this stage.

[1]Mehmet AkifErsoy University Faculty of Education, Mathematics Education Department,15100 Burdur/Turkey.

Exact definitions of geometric figures can be made. They can make such deductions as "If one angle of a parallelogram is right, then the other angles are also right." but they cannot prove it mathematically. Children can think properties separately from the whole at this stage. Students' observations at this level go beyond figure properties and start to focus on logical discussions about properties. At this level, students can understand informal deductive discussions about figures and their properties.

Fuys et al. (1988) listed the characteristics of this level as follows: Students are able to

- Use certain geometric properties to define a class of geometric figures and test if these properties are sufficient.
- Identify the minimum properties required to define a geometric figure.
- Use definitions and formulas for a group of figures.
- Make deductions over the information given and prove the deduction using logical relationships.
- Order geometric figures.
- Follow a proof and make suggestions about the steps.
- Express and summarize a proof in their own words.
- Make more than one explanations to prove one thing and try to confirm this using diagrams.
- Understand the difference between a premise and its opposite informally.
- Use strategies and reasoning in problem solving.
- Understand deductive expressions and approach problems in this way of thinking.
- Students cannot comprehend the meaning of deduction axiomatically (cannot see the need for postulates and pre-propositions).

According to Murray (1997), students can use definitions when reasoning about figure properties and understand the interrelations both among figure properties and figures. For example, they can make such deductions as "Since opposite sides of a parallelogram are parallel, its opposite angles are equal as well". Students can use informal reasoning like "If… ,then …".

For a student at Level 3 (formal deduction), properties of shapes become objects that are independent of the figure and the object itself. This stage corresponds to high school years. Students can use axiomatic structure at this stage and can construct proofs themselves within this system. Thinking at this stage is related to deduction according to the reverse of a theorem, axioms and necessary and sufficient conditions. Thinking objects at Level 3 are the relationships among the properties of objects. A system structure starts to develop with axioms, definitions, theorems, results and assumptions. Students can work with abstract expressions about geometric properties and can make deductions based on logic rather than intuition. Students at Level 3 prove other theorems with deduction using previously proven theorems and axioms, and can achieve reasoning processes through induction. They can recognize two different logical reasoning ways with the same theorem and differentiate between them.

Students at Level 4 can see the relationships and differences between two different axiomatic systems. The thinking objects of this level are "deductive axiomatic systems for geometry." They suggest theorems in different axiomatic systems and analyze and compare these systems.It is known that Van hielelevels depend on experience rather than age. What kind of experiences the students have experienced can be determined through the curriculum. It is known that 11th graders are at level 3. This study aimed to determine 11th graders' levels of van hiele geometric thinking. In this respect, it is assumed that the findings of the current research will contribute the works of construction of the geometry curriculum.

On the other hand, my literature research indicates that there has been little investigation on the Van Hiele levels of understanding of high school students in Turkey. Therefore, in order to inform any major revision of the high school curriculum, it would seem necessary first to determine the van Hiele geometric thinking levels of high school students. The findings of the studies revealed that the students' geometric thinking level were below the expected level. Baffoe and Mereku (2010) stated that 59% of the senior high school students in Ghana were Van Hiele level 1. Alex and Mammen (2012) found that majority of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the Van Hiele Theory were at level 0. The study that was conducted by Abu and Abidin (2013) categorized the students as 90 students from level 0, 60 students from level 1, and 30 students from level 2. Genz (2006) found that high school students were not adequately prepared to understand the concepts of geometry. Çakmak&Güler (2014) determined that the mathematics teacher candidates' level of Van Hiele were generally level 3. At the end of the research by Vojkuvkova&Haviger (2013); it was determined that 96.7 % of Czech Secondary School students achieved Level 1, 86.5 % achieved Level 2, 39.1 % achieved Level 3 and 8.8 % achieved Level 4. Bal (2011) tarafındanyapılançalışmada, it was seen that the teacher candidates are at the "3: Informal Deduction/Order" level the most (33.6 %) and they are at the "5-Being able to see the relationships (Rigor)" level the least (2.2 %). It was also found out that 22.6 % of the teacher candidates were at level "0" and they couldn't be assigned into any levels.

II. METHOD

The aim of the present study is to investigate Van Hiele stage 2 geometric thinking levels of 11th grade students. A total of 50 students from two 11th grade classes, which were selected randomly from two high schools in Antalya, participated in the study. 28 of the students are girls and 22 are boys. 24 of the participants are regular high school students and 26 students are from Anatolian high school. Data was collected asking a total of 12 questions consisting of 4 open-ended questions (questions 1-4) and 8 multiple choice questions (questions 5-12) appropriate for the Van Hiele stage 2 thinking level. The questions were designed within the scope of "figure definitions (1st question)", "figure properties (2nd and 4th questions)", "relationships among figures" (3rd question), "relationships among properties" (5th-12th questions). Questions were designed together with two mathematic educators. The present study was carried out in a quantitative method. The answers to the open-ended questions were examined through descriptive analysis method. The percentages of the students' correct answers are presented and incorrect answers were also analyzed and mistakes were found out. Findings obtained from the multiple choice questions, on the other hand, are presented on frequency and percentage tables.

III. FINDINGS

Findings Regarding Question 1

In the first question, the students were asked to write the definition of the square, rectangle, parallelogram, rhombus, trapezoid and a deltoid. The findings obtained are presented on Table 1.

Table 1. Frequency and percentages regarding definitions of the figures

	Correct	Wrong	Unanswered
Square	36(72%)	11(22%)	3(6%)
Rectangle	28(56%)	18(36%)	4(8%)
Parallelogram	34(68%)	9(18%)	7(14%)
Rhombus	34(68%)	4(8%)	12(24%)
Trapezoid	21(42%)	6(12%)	23(46%)
Mean	31(62%)	10(20%)	9(18%)

Incorrect definitions made by the students are as follows:

- Square: "A quadrangle with equal sides (9 students)", "A polygon having four sides and adjacent sides of which are perpendicular (1)", "A quadrangle whose all sides are equal in length with right diagonals crossing centering each other (1)"
- Rectangle: "A quadrangle whose opposite angles are equal (9)", "A quadrangle whose opposite sides are equal (8)", "A quadrangle whose opposite sides and angles are equal to each other (1)".
- Parallelogram: "A quadrangle whose opposite sides are equal (6)", "A quadrangle whose opposite angles having a sum of 180 degrees (2)", "A quadrangle whose opposite angles are equal (1)".
- Rhombus: "A quadrangle whose all sides and angles are equal to each other (1)", "A quadrangle whose all angles are equal (3)"
- Trapezoid: "A shape whose upper and lower sides are equal (1)", "A shape whose all sides are different (1)", "A shape whose all sides are irregular (3)", "A shape whose opposite angles are equal (1)".

With regard to level 2, the students are expected to be able to express the properties of the geometric figures. However, when analyzed the wrong answers; it was seen that 20% of the participants did not know the minimum properties determining a class of geometric shape.Besides, the fact that the relationships between geometric properties are not known is a factor affectind this result. For example, while defining the square, the students assume that"if four edges are equal then four angles are equal" or"if the opposite edges of a quadrangle are equal then the angles are equal" while defining the rectangle.

Findings Regarding Question 2

In this question, the students were given a geometric property and were asked about which geometric property is caused by this given property. When looked in terms of Van Hiele geometric thinking levels; a student in the level 2 is expected to know thw relationships between geomwtric properties.The 2nd question asked students "If diagonals center each other, is this shape a parallelogram? Why?" While 37(74%) of the students answered the question correctly, 3 (6%) students gave incorrect answers and 10(20%) of them did not answer the question. Incorrect answers contained the following mistakes: "Shapes like square, rhombus center (2)", "Not all shapes that center are parallelograms (1)".

Findings Regarding Question 3

In this question, the students are expected to know which quadrangle is a subset of another one using the properties of quadrangles. The findings obtained are presented on Table 2.

Table 2. Frequency and percentages regarding the knowledge of ordering figures

	Correct	Wrong	Unanswered
A) Every square is a rhombus.	42(84%)	2(4%)	6(12%)
B) Every parallelogram is trapezoid.	16(32%)	7(14%)	27(54%)
C) Every rectangle is parallelogram.	42(84%)	1(2%)	7(14%)
Mean	33(66%)	3(6%)	13(26%)

2 students who answered the item A incorrectly wrote "The angles are not equal". In the wrong answers in the item B, it was seen that 6 students said "There is no parallelism" while 1 student wrote "Trapezoids can be different". 1 student who answered the item C incorrectly said "Because its opposite sides are equal to each other".With regard to level 2, the students are expected to be able to order the classes of geometric shapes.

Findings Regarding Question 4

In this question, the students were asked to write the quadrangles whose diagonals intersect at right angles and center each other. 24(48%) students gave correct answers to this question whereas 23(46%) students answered incorrectly and 3(6%) of them did not answer the question. In the analyses of the mistakes, the following answers were found: square and deltoid (6), square and rectangle (9), square, deltoid and rectangle (4), deltoid (4).With regard to level 2, the students are expected to know the geometric properties determining the classes of the geometric shapes.

Findings Regarding Questions 5-12

In the multiple choice questions, the students were given a geometric property and were asked about which geometric property is caused by this given property. As known,"relationships among geometric properties" is among thinking products of Van Hiele geometric thinking level 2.Frequencies and percentages of the correct answers are presented on Table 3.

Table 3. Frequency and percentages regarding the knowledge of interrelations between geometric properties

Item	Frequency	Percentage
5	32	64%
6	24	48%
7	28	56%
8	18	36%
9	25	50%
10	28	56%
11	31	62%
12	40	80%
Mean	28	56%

IV. DISCUSSION, CONCLUSION AND RECOMMENDATIONS

Students' achievement rate in defining geometric figures was found to be 62% on average. The examination of the common mistakes showed that a square was defined as "a quadrangle whose sides are equal to each other", a rectangle as "a quadrangle whose opposite angles are equal to each other", a parallelogram as "a quadrangle whose opposite sides are equal", a rhombus as "a quadrangle whose all angles are equal" and a trapezoid as "a shape whose all sides are irregular/different". When the mistakes were analyzed, it was seen that the students did not write some of the geometric properties that define a certain shape while making their definition. In this process, it could be asserted that the students failed to know "the minimum properties that define a figure".In the questions regarding whether a given geometric property is descriptive for a class of geometric figures; achievement rate was found as 61%. In the analyses of the mistakes, it was observed that instead of showing which group of geometric figures can be defined with the given geometric property in a logical way, the students assessed the given property looking at the appearance of geometric shape. In other words, they took the appearance of the shape into consideration. For instance, in the question asking which shapes have diagonals intersecting at right angles, instead of drawing a quadrangle with its diagonals intersecting at right angles and concluding on whether this property would require the angles of the quadrangle to be equal through geometric thinking, the students drew a rectangle and showed whether the diagonals intersected at right angles visually. In this process, just like drawing a deltoid like a rhombus, for example, wrong drawing of the appearance of the shape or drawings in a specific form affected the results negatively as well. Secondary school students are supposed to consider the properties of shapes independently of the shapes themselves (Altun, 2000; Toluk et al., 2002; Duatepe, 2001). In the study carried out by Ubuz (1999), the main reason for the mistake that 10th and 11th grade students make in geometry was found to be formulation of ideas based on the appearance of a figure. Kesici (2005) concluded that high school students associated properties of shapes to their appearances. In addition, it was observed that overall the students' were not at the levels they were expected to be. In questions about ordering figure, 66% of the students could give the correct answer. In the analyses of the wrong thinking; it was concluded that the students thought as follows: "Since at least one property of B is not always true for A, A⊄B. For example, as the opposite sides of a trapezoid are not parallel, a parallelogram was considered not to be a trapezoid. However, a parallelogram is a trapezoid because its two opposite sides are parallel. In this respect, students' mistakes in ordering figures can be explained with two reasons. The first one is the lack of information in defining figures and the second one is failure to comprehend the logic "if A has the properties of B, then A⊂B". In this case, it is the failure to comprehend the logic that each A is a B at the same time. For instance, "a quadrangle with equal side lengths is called a rhombus", "a quadrangle with equal side lengths and angles is called a square". If we represent the set of all rhombuses as E and the set of all squares as K, then K⊂E. In this case, "Each square is a rhombus at the same time".

In the eight multiple choice questions, the students were given a geometric property and were asked this given property cause which geometric property. The average achievement rate in these questions was found to be 56%. Achievement in these questions is evaluated by the ability to find out the stated specific geometric property by drawing a quadrangle suitable with the given geometric property through some geometric thinkings. Considering a certain quadrangle and looking at this property in this specific figure instead of thinking over a general quadrangle possessing the given geometric property may cause to obtain incorrect results. On the other hand, in the case that starting with a general quadrangle drawing but making decisions by looking at the shape rather than using logical thinking or wrong geometric thinking may be the source of mistakes.

In the Van Heile geometric thinking levels, 11th graders are expected to be at the 3rd level (deduction stage). The products of this though level include; "reasoning through induction", "can construct proof", "can prove other theorems with deduction by using theorem and axioms", " can differentiate between a statement and its opposite". The products of the thought level 2 on the other hand are as follows: "formal figure definitions can be understood", "can understand relationships within and among shapes", "has the knowledge to order the shapes", "knows the properties of the shapes and the relationships among these properties", "can identify the minimum properties required for defining a geometric shape" (Olkun and Toluk 2003; Crowley 1987). Therefore, 11th graders are expected to answer the questions in this test correctly. However, the findings obtained show that students performed under 61% on average at the 2nd stage of Van Hiele geometric thinking levels. In other words, most of the students are not the level they are expected to be. Several other studies have revealed similar findings. In a study carried out by Oral, İlhan and Kınay (2013), it was determined that 8th grade students were at the 1st level of geometric thinking and were accumulated at level 0 in terms of algebraic thinking. The study also found a positive, moderate and significant relationship between students' geometric and algebraic thinking levels. Mayberry (1981, 1983) studied on the five levels of geometric thought hypothesized by Van Hiele. The results showed that many pre-service teachers who took high school geometry classes were under level 3. Most of the teachers could neither perceive the properties of figures in stage 1 characteristic nor could they conceive the properties within figures and the properties they contained in stage 2 characteristic. In another study conducted by Duatepe and Akkuş (2003) candidate pre-school teachers' Van Hiele thinking levels were found to be rather low. Moreover, the study found that pre-service teachers who graduated from vocational high schools had lower geometric thinking levels than those who graduated from other high schools. Çetin and Dane (2004) found that 65% of the pre-service elementary teachers could not recognize and practice basic concepts taught in geometry. In the study conducted by Şahin (2008), 74% of the pre-service elementary teachers and 86% of the elementary teachers were determined to be under stage 3 of Van Hiele thinking levels.

Duatepe's (2000) study suggested that half of the elementary teachers were at level 2 and 29% of them were at level 3. Similarly, Olkun, Toluk and Durmuş (2002) found that pre-service elementary teacher's Van Hiele geometric thinking levels were at level 1 (visualization) for 23%, Level 2 (analysis) for 41% and level 3 (ordering) for 26%. In a similar study conducted by Toluk, Olkun and Durmuş (2002) experience was observed to ineffective on Van Hiele thinking levels of pre-service elementary teachers. Several studies have been carried out to improve students' geometric thinking. In the study of Güven (2006) determined that drawing practices in geometry developed students' Van Hiele geometry understanding levels. Breen (1999) showed discovery based activities in environments equipped with Windows TM Geometry aided computers helped 8th graders to get to the 2nd and 3rd levels of geometric thought of Van Hiele. Moreover, computer aided teaching practiced using computer software such as dynamic geometry,Logo or geometrylearning video based on Van HieleTheoryhas positive effects on students' (Abu &Abidin, 2013; Frerking, 1994;Scally, 1991). On the other hand, Aksu (2005) stated that active learning in primary education wass more effective than traditional teaching in increasing student achievement in geometry.

REFERENCES

[1] Abu, M. S.,& Abidin, Z. Z. (2013). ImprovingtheLevels of geometricthinking of secondaryschoolstudentsusinggeometrylearning video based on Van HieleTheory. International Journal of Evaluation andResearch in Education, 2(1), 16-22.

[2] Aksu, H. (2005). İlköğretimdeaktiföğrenmemodeliilegeometriöğretimininbaşarıyakalıcılığa, tutumavegeometrikdüşünmedüzeylerineetkisi. (YayınlanmamışDoktoraTezi). DokuzEylülÜniversitesi, İzmir.

[3] Alex, J. K.,&Mammen, K. J. (2012). A survey of South Africangrade 10 learners' geometricthinkinglevels in terms of the Van HieleTheory. Anthropologist, 14(2), 123-129.

[4] Altun, M. (2000).Matematiköğretimi. Bursa: Alfa Yayınları.

[5] Assaf, S. A. (1986). The effects of using logo turtle graphics in teaching geometry on eight grade students' level of thought, attitude toward geometry and knowledge of geometry. Dissertation Abstract Index, 46 (10), 2925A.

[6] Baffoe, E. &Mereku, D. K. (2010). The Van Hielelevels of understanding of studentsenteringseniorhighschool in Ghana. AfricanJournal of EducationalStudies in MathematicsandSciences, 8, 51-61.

[7] Bal, A. P. (2011). Geometrythinkinglevelsandattitudes of elementaryteachercandidates. İnönü UniversityJournal of theFaculty of Education, 12(3), 97-116.

[8] Breen, J. J. (1999). Achievement of Van Hiele level two in geometry thinkig by eight grade students through the use of geometry computer-based guided instruction. (Unpublished Doctoral Dissertation).The University of South Dakota, Dakota.

[9] Burger, W. F., & Shaughnessy, J. M. (1986).Characterizing the Van Hiele levels of development in geometry.Journal for Research in Mathematics Education, 17, 31-48.

[10] Çakmak, D., &Güler, H. K. (2014).İlköğretimmatematiköğretmeniadaylarının geometric düşünmedüzeylerininbelirlenmesi.Türk Eğitim Bilimleri Dergisi, 12(1), 1-16.

[11] Crowley, M. L. (1987). The Van Hiele model of the development of geomemc thought. In M. M. Lindquist (ed.), In learning and teaching geometry, K-12, 1987 yearbook of the National Council of Teachers of Mathematics, (pp.1-16). Reston, VA: National Council af Teachers af Mathematics.

[12] Çetin, Ö. F., & Dane, A. (2004).Sınıföğretmenliği III.sınıföğrencileriningeometrikbilgilereerişidüzeyleriüzerine. KastamonuEğitimDergisi, 12(2), 427–436.

[13] De Villiers, M. D., &Njisane, R. M. (1987).The development of geometric thinking among black high school pupils in Kwazulu (Republic of South Africa). In J. C. Bergeron, N. Herscovics, & C. Kieren (eds.), Proceedings of the 11th conference of the international group for the psychology of mathematics education (pp. 117-123). Montreal.

[14] Duatepe, A. (2000). İlköğretimöğretmenadaylarının Van Hielegeometrikdüşünmedüzeylerininbelirlenmesi. VI Fen BilimleriEğitimiKongresi, 6-8 Eylül, HacettepeÜniversitesi, Ankara.

[15] Duatepe, A. (2001, Eylül).Van Hielegeometrikdüşünmeseviyeleriüzerinenitelikselbiraraştırma.IV. Fen BilimleriKongresi. Ankara.

[16] Duatepe, A., &Akkuş Ç. (2003).OkulöncesiÖğretmenadaylarının Van Hieledüşünmeseviyelerininbelirlenmesi. OMEP 2003 DünyaKonseyToplantısıBildiriKitabı. Kuşadası.

[17] Frerking, B. G. (1994). Conjecturing and proof-writing in dynamic geometry.Dissertation Abstract International, 55, 12.

[18] Fuys, D., Geddes, D., Lovett, C. J., &Tischler, R. (1988).The Van Hiele model of thinking in geometry among adolescents.Journal for Research in Mathematics Education. Reston, VA: NCTM.

[19] Genz , R. (2006). Determining High School GeometryStudents ' GeometricUnderstanding Using Van HieleLevels: Is There a DifferenceBetweenStandards-basedCurriculumStudentsandNonStandards-BasedCurriculumStudents? (Unpublished Master Thesis). BrighamYoungUniversity.

[20] Güven, Y. (2006). Farklıgeometrikçizimyöntemler, kullanımınınöğrencilerinbaşarı, tutumve Van Hielegeometrikanlamadüzeylerineetkisi. (YayımlanmamışYüksekLisansTezi). KaradenizTeknikÜniversitesi, Trabzon.

[21] Van Hiele, P. M. (1986). Structure and insight: A theory of mathematics education. Orlando: Academic Press.

[22] Hoffer, A. (1983). Van Hiele-Based Research. In R. Lesh, & M. Landau (eds.) Acquisition of mathematics concepts and processes.Orlando, Florida: Academic Press, INC.

[23] Kesici, A. (2005). Liseöğrencilerinin geometri-1 dersindegeçenbazıkavramlarıöğrenmedüzeyleriüzerinebiraraştırma. (YayımlanmamışYüksekLisansTezi). YüzüncüYılÜniversitesi, Van.

[24] Kılıç, Ç. (2003). İlköğretim 5.sınıfmatematikdersinde Van Hieledüzeylerinegöreyapılangeometriöğretimininöğrencilerinakademikbaşarıları, tutumlarıvehatırdatutmadüzeyleriüzerindekietkisi. (Yayımlanmamışyükseklisanstezi).AnadoluÜniversitesi, Eskişehir.
[25] Mayberry, J. (1981). An investigation in the van Hiele levels of geometric thought in undergraduate pre-service teachers. Disserations Abstract International, 42, 2008A. (UMI No. 80-23078).
[26] Mayberry, J. (1983) The Van Hiele levels of geometric thought in undergraduate preservice teachers. Journal for Research in Mathematics Education, 14, 58-69.
[27] Murray, J. C. (1996). A generalised Van Hiele.(Unpublished manuscript).University of Stellenbosch, Stellenbosch.
[28] Murray, J. C. (1997). The Van Hiele theory. Paper presented at the MALATI / EMSCEP Geometry Thinkshop, University of Stellenbosch, Stellenbosch, South Africa.
[29] Olkun, S., &Toluk, Z. (2003).Matematiköğretimi. Ankara: AnıYayıncılık.
[30] Olkun, S., Toluk, Z., &Durmuş, S. (2002, Eylül). Matematikvesınıföğretmenliğibirincisınıföğrencileriningeometrikdüşünmedüzeyleri. V. Ulusal Fen BilimleriveMatematikEğitimiKongresi'ndesunulmuşsözlübildiri. OrtadoğuTeknikÜniversitesi, Ankara.
[31] Oral, B., İlhan, M., &Kınay, İ. (2013). 8. sınıföğrencileriningeometrikvecebirseldüşünmedüzeyleriarasındakiilişkininincelenmesi. PamukkaleÜniversitesiEğitimFakültesiDergisi, 34(2), 33-46.
[32] Scally, P. S. (1991).The impact.Dissertation Abstract Index, 52(3), 372 A52 (3), 372 A.
[33] Senk, D. L. (1983).Proof-writing achievement and Van Hiele levels among secondary school geometry students.Dissertation Abstract Index.44(02) 417A.
[34] Şahin, O. (2008).Sınıföğretmenlerininvesınıföğretmeniadaylarının Van Hielegeometrikdüşünmedüzeyleri. (YayımlanmamışYüksekLisansTezi). AfyonKocatepeÜniversitesi, Afyon.
[35] Toluk, Z., Olkun, S., &Durmuş, S. (2002, Eylül). Problem merkezlivegörselmodellerledestekligeometriöğretimininsınıföğretmenliğiöğrencileriningeometrikdüşünmedüzeyleriningelişimineetkisi.5. Ulusal Fen BilimleriveMatematikEğitimiKongresi'ndesunulmuşbildiri. ODTÜ, Ankara.
[36] Tutak, T., &Birgin, O. (2008, Mayıs).Dinamikgeometriyazılımıilegeometriöğretimininöğrencilerin Van Hielegeometrianlamadüzeylerineetkisi.8. UluslararasıEğitimTeknolojisiKonferansı'ndasunulmuşsözlübildiri.Eskişehir, AnadoluÜniversitesi.
[37] Ubuz, B. (1999). 10. ve 11. sınıföğrencilerinintemelgeometrikonularındakihatalarıvekavramyanılgıları. HacettepeÜniversitesiEğitimFakültesiDergisi, 16(17), 95-104
[38] Usiskin, Z. (1982). Van Hiele levels of achievement in secondary school geometry: CDASSG Project, Chicago University.
[39] Van de Walle, J. A. (2004). Elementary and middle school mathematics: teaching developmentally. New York: Pearson Education, Inc.
[40] Vojkuvkova, I., &Haviger, J. (2013).The Van Hielegeometry test at Czechsecondaryschool. WDS'13 Proceedings of ContributedPapers, Part I, 112–115.